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ABSTRACT

This guidebook specifies minimum course content covering number systems and bases, rational numbers and operations with rationals, and solving simple open sentences. Course goals are stated, then performance objectives, a course outline, references, and suggested teaching strategies are listed for each topic covered. Posttest items and a list of 15 references are included. (DT)

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AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

Pure Mathematics 3
#5211.33

Mathematics

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QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

PURE MATHEMATICS 3

5211.33

(EXPERIMENTAL)

Written by

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and
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for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been editied by a subcommittee of the Mathematics Advisory Committee.

GOALS

1. To help the student discover the value of the Hindu-Arabic system through study of other number systems and bases.

2. To develop the student's understanding of the properties of rational numbers and to improve his skill in operations with rationals.

3. To develop the student's ability to solve simple open sentences by applying the properties of equality.

KEY TO REFERENCES

(* State adopted)

K(1) Keedy, Mervin; Jameson, Richard; and Johnson, Patricia. Exploring Modern Mathematics, Book 1. New York: Holt, Rinehart and Winston, Inc., 1963.

K(2) Keedy, Mervin; Jameson, Richard; and Johnson, Patricia. Exploring Modern Mathematics, Book 2. New York:

Mc(7) McSwain, E. T.; Brown, K. W.; Gundlach, B. H.; and Cooke, R. J. Mathematics 7. River Forest, Illinois: Laidlaw Brothers, 1965.

Mc(8) McSwain, E. T.; Brown, K. W.; Gundlach, B. H.; and Cooke, R. J. Mathematics 8. River Forest, Illinois: Laidlaw Brothers, 1965.

N-PA Nichols, Eugene D. Pre-Algegra Mathematics. New York: Holt, Rinehart and Winston, Inc., 1965.

*SMS(1) Suppes, Patrick; Meserve, Bruce; Sears, Phyllis. Sets, Numbers, and Systems 1. Brandon, Miss.: The L. W. Singer Company, Inc., 1969.

*SMS(2) Suppes, Patrick; Meserve, Bruce; Sears, Phyllis. Sets, Numbers, and Systems 2. Brandon, Miss.: The L. W. Singer Company, Inc., 1969.

PERFORMANCE OBJECTIVES

The student will

1. Name several ancient numeration systems.
2. Change from Roman numerals to decimal numbers.
3. Change from decimal numerals to Roman numerals
4. Construct a place value table for any base to five places.
5. Expand a numeral in any base.
6. Change a five digit numeral of any base to a decimal numeral.
7. Change any decimal numeral, 3000 or less, to any other base.
8. Add and subtract numerals of other bases.
9. Find the product of three place numerals of any base.

COURSE OUTLINE

I. Numeration Systems

- A. History
  - 1. Compare ancient systems to Hindu-Arabic system.
- B. Roman Numerals
  - 1. Identify symbols as Roman numerals
  - 2. Change from Roman numerals to Hindu-Arabic and vice-versa
- C. Decimal System
  - 1. Review exponential notation.
  - 2. Place Value
  - 3. Expanding numerals using exponential notation.
- D. Bases Other Than Ten
  - 1. Place Value
  - 2. Expanding Numerals
  - 3. Changing to base 10
  - 4. Changing from base 10 to other bases
  - 5. Computation
    - a. Addition
    - b. Subtraction
    - c. Multiplication

REFERENCES

K(1) (pp. 1-34) Uses discovery approach. Follows the outline for sequence.

Mc(7) (pp. 313-332) Follows the outline for sequence. Gives explanation then exercises.

Mc(8) (pp. 5-21) Reviews extensively supplementary work.

N-PA (pp. 1-63) Same as Mc(7) but graphically shows the relationship with computers.

SMS (1) (pp. 68-81) Contains good explanations but practices too short.

STRATEGIES

1. Let the student find how tedious ancient systems were by doing some selected problems.
2. Let the students try some multiplication with Roman numerals to find how cumbersome it is.
3. Give a comprehensive review of base ten and show that other bases are structured the same.
4. It is important to point out the difference between the number systems and numeration systems.

PERFORMANCE OBJECTIVES

The student will

1. Recognize elements of subsets of the rational numbers.
2. Order a given set of rational numbers.
3. Perform the four operations with rational numbers.
4. Identify the commutative, associative, distributive, inverse, identity, and closure properties of rational numbers, when illustrated by a specific example.

COURSE OUTLINE

II. Rational Numbers

A. Number Systems

1. Review of definitions and abbreviations.

a. Natural numbers (N){1, 2, 3...}

b. Whole numbers (W){0, 1, 2...}

c. Arithmetic numbers (any number that can be expressed as the ratio of a whole number to a natural number)

d. Integers (I) {...,-1,0,1,...} or {natural numbers, their additive inverses, and zero}

2. Rational numbers (Q)

a. Definition (any number that can be expressed as the ratio of an integer to a natural number) or terminating decimal)

B. Ordering Rationals

C. Operations

1. Addition
2. Subtraction
3. Multiplication
4. Division

D. Properties

1. Commutative
2. Associative
3. Distributive
4. Inverse
5. Identity
6. Closure

REFERENCES

K(2) (pp. 162-200) Studies the rational numbers as a number system with definitions, order, properties, and operations.

N-PA (pp. 139-144, 222-246) Refers only to positive rational but moves to "directed real numbers" with the operations and properties.

SMS(2) (pp. 96-116, 136-148) Reviews rationals then introduces the irrationals with square root and the real number.

SUGGESTED STRATEGIES

1. Arithmetic numbers are used to name the non-negative rationals to avoid confusion since they are studied before the concept of positive and negative is introduced. When the students understand the rational numbers there will no longer be a need for the terminology and it can be eliminated.

2. While investigating the definition of rational numbers and repeating decimals, it is a good time to introduce irrational numbers {non-terminating, non-repeating decimals} or {any number that can <u>not</u> be expressed as the ratio of an integer and a natural number} and the real numbers (Q, Ir).

3. To develop the concept of order, use the number line and locate points whose coordinates are tational numbers.

4. Since the operations with integers were covered thoroughly in Pure Math 1, the student should know the rules. He also knows that I Q so it would be contradictory to have different rules.

5. To clarify the commutative properties, have the student specify whether it is the <u>addends</u> or the <u>factors</u> which are being commuted; for example,

   $(5+3) \times 2 = 2 \times (5+3)$ and $(5+3) \times 2 = (3+5) \times 2$.

6. If time permits, give an overview of clock arithmetic to really show the student the meaning of mathematical system.

PERFORMANCE OBJECTIVES

The student will

1. Translate simple word phrases and sentences into symbols.
2. Identify or write examples of the reflexive, symmetry, and transitive properties of equality.
3. Use the addition and multiplication properties of equality and the distributive property of rational numbers to solve equations.
4. Name the properties justifying successive steps of simple proofs.
5. Solve verbal problems by writing an equation and finding its solution set.

COURSE OUTLINE

III. Open Sentences

- A. Translation from words to symbols.
- B. Properties of Equality
  1. Reflexive
  2. Symmetric
  3. Transitive
  4. Substitution
- C. Solving Equations
  1. Using the addition property.
  2. Using the multiplication property.
  3. Combining the addition and multiplication properties.
  4. Using the distributive property to combine similar terms.
  5. Using the addition property when the variable appears in both members of the equation.
- D. Proofs
  1. Introduction
  2. Supplying the reasons
- E. Verbal Problems
  1. Writing the equation
  2. Solving the equation
  3. Checking the answer

REFERENCES

K(2) (pp. 285-309) Contains complete coverage of solving equations using rational numbers as the domain.

Mc(7) (pp. 33-51) Has introductory work for problem solving.

Mc(8) (pp. 61-95) Covers the material well but does not cover the reflexive, symmetric, and transitive properties.

N-PA (pp. 401-410) Includes useful information and numerous examples of translating words to symbols. Uses "cover-up" method in solving equations.

SMS(1) (pp. 101-102, 170-179) Develops additon and multiplication properties of equality and translating words to symbols. No mention of reflexive, symmetric, and transitive properties.

SMS(2) (pp. 172-183) Reviews material in Book 1 and further development.

SUGGESTED STRATEGIES

1. The more able students will discover the properties of equality and be able to state them informally.

2. It is suggested that only integers be used in introducing open sentences and proofs. After the concepts are learned, the operations with rationals may be reviewed by including rationals in the equations.

3. When students are learning to solve equations it is sometimes helpful to them to indicate the property they are using in each step of the solution.

   Example:

   $2x + 3 = 7$

   $A(-3)$ $\quad 2x = 4$

   $M(\frac{1}{2})$ $\quad x = 2$

4. Students readily accept the truth of the reflexive, symmetric and transitive properties but have difficulty remembering the names.

5. The symmetric property of equality and the commutative property are often confused. Point out that the symmetric property "reverses" an equation and the commutative property "reverses" an operation.

SAMPLE POSTTEST ITEMS

I.

1. List two ancient numeration systems.
2. Change the following Roman Numerals to decimal notation:
   a. XIV
   b. CLXXIX
   c. XLVI
3. Express these decimal numerals as Roman numerals:
   a. 56
   b. 1980
   c. 139
4. Construct a base seven place value chart to five places.
5. Write 43,271.25 in expanded notation using powers of ten.
6. Change this to a decimal numeral: 10425_6.
7. Change this decimal numeral to a base four numeral: 768.
8. Perform the operations in the indicated bases:
   a. $312_5 + 434_5$
   b. $513_6 - 245_6$
   c. $110_2 + 111_2$
9. Find the product of 423_5 and 102_5.

II.

1. Indicate which statements are true and which are false.
   a. $5 \in Q$
   b. $-\frac{1}{2} \in U$
   c. $0 \in Q$
   d. $I \subset W$
   e. $I \subset Q$
   f. $-3 \in N$

2. Arrange the numbers from smallest to largest.

   $-1,\ 2.4,\ .3,\ -3,\ -\frac{5}{2},\ \frac{5}{6}$

3. Perform the indicated operations. Put all results in simplest form:

   a. $\frac{1}{2} - \frac{1}{5} =$

   b. $100 - 0.75 =$

   c. $6 - (\frac{3}{4}) =$

   d. $-4.2 \times 0.06 =$

   e. $-\frac{2}{3} \div \frac{4}{5} =$

   f. $\frac{1}{2} \times \frac{1}{2} =$

   g. $29 \times 0.01 =$

   h. $-6.4 \div 0.2 =$

   i. $-\frac{3}{4} \times -\frac{5}{5} =$

   j. $\frac{3}{7} \times \frac{2}{21} =$

4. Identify the property illustrated:

   a. $-7 \times 8 = 8 \times -7$

   b. $a \cdot 1 = a$

   c. $(a + b) + c = a + (b + c)$

   d. $\frac{1}{2}\ (\frac{1}{3} + \frac{2}{5}) = \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{5}$

   e. $-9 + 0 = -9$

   f. $(x\ y)\ z = x\ (y\ z)$

III.

1. Using numerals and variables express the following as number phrases or sentences:

   a. Some number plus eight equals fifteen.

b. Nine more than a number.

c. Five times a number is forty.

d. Six less than twice a number.

2. Use variables a, b, c, d to illustrate these properties of equality.

a. The transitive property

b. The symmetric property

c. The reflexive property

3. Solve, check and give solution sets. Show each step of the solution. The domain is $\{$ Integers $\}$.

a. $3x + 4 = 13$

b. $\frac{N}{4} = -12$

c. $5c - 6 = 4$

d. $b + 8 = 2$

e. $\frac{3y}{7} + 3 = 12$

f. $2x + 35 + 9x = 2$

4. For each lettered step name the property which justifies that change from the preceeding statement.

a.
$$\begin{aligned} n - 6 &= 19 \\ n - 6 + 6 &= 19 + 6 \quad (a) \\ n &= 25 \quad (b) \end{aligned}$$

b.
$$\begin{aligned} 7 \times (4 \times \tfrac{1}{7}) &= 7 \times (\tfrac{1}{7} \times 4) \quad (a) \\ &= (7 \times \tfrac{1}{7}) \times 4 \quad (b) \\ &= 1 \times 4 \quad (c) \\ &= 4 \quad (d) \end{aligned}$$

c. For each r

$$\frac{3r + 15}{3} = 1 \quad (3r + 15)$$

$$= \frac{1}{3}(3r) + \frac{1}{3}(15) \quad \text{(a)}$$

$$= \left(\frac{1}{3} \cdot 3\right) r + \frac{1}{3}(15) \quad \text{(b)}$$

$$= 1 \cdot r + 5 \quad \text{(c)}$$

$$= r + 5 \quad \text{(d)}$$

5. Show the complete solution to each of the following problems:

   a. The sum of twice a number and 16 is 86. What is the number?

   b. If the weight of a cubic foot of gold is miltiplied by 2 and 196 poinds is subtracted from the product, the result is 1 ton. How nuch does a cubic foot of gold weigh?

POSTTEST ANSWER KEY

I.

1. Answers will vary.
2. a. 74
   b. 179
   c. 46
3. a. LVI
   b. MCMLXXX
   c. CXXXIX
4.

7^4	7^3	7^2	7^1	7^0
2401	343	49	7	1

5. $4 \cdot 10^4 + 3 \cdot 10^3 + 2 \cdot 10^2 + 7 \cdot 10^1 + 1 \cdot 10^0 + 2 \cdot 10^1 + 5 \cdot 10^2$
6. 1457
7. 30000_4
8. a. 1301_5
   b. 12023_6
   c. 1101_2

II.

1. a. T
   b. F
   c. T
   d. F
   e. T
   f. F

2. $-3, -\frac{5}{2}, -1, .3, \frac{5}{6}, 2.4$

3. a. $\frac{3}{10}$

   b. 99.25

   c. $6\frac{3}{4}$

   d. .252

   e. $-\frac{5}{6}$

   f. $\frac{1}{4}$

   g. .29

   h. -32

   i. $-\frac{3}{4}$

   j. $\frac{11}{21}$

4. a. commutative, multiplication

   b. identity, multiplication

   c. associative, addition

   d. distributive

   e. identify, addition

   f. associative, multiplication

III.

1. a. $n + 8 = 15$

   b. $r + 9$

   c. $5x = 40$

   d. $2x - 6$

2. a. If $a < b$ and $b < c$, then $a < c$.

b. If $a + b = c$, then $c = a + b$

c. $a + a$

3. a. $3x + 4 = 13$

$3x = 9$

$x = 3$

$\{3\}$

b. $\frac{N}{4} = -12$

$N = -48$

$\{-48\}$

c. $5c - 6 = 4$

$5c = 10$

$c = 2$

$\{2\}$

d. $b + 8 = 2$

$b = -6$

$\{-6\}$

e. $\frac{3y}{7} + 3 = 12$

$\frac{3y}{7} = 9$

$y = 21$

$\{21\}$

f. $2x + 35 + 9x = 2$

$11x + 35 = 2$

$11x = 33$

$x = -3$

$\{-3\}$

4. a. (a) addition property of equality

(b) additive inverse

b. (a) commutative

(b) associative

(c) multiplicative inverse

(d) multiplicative identity

c. (a) distributive

(b) associative

(c) multiplicative inverse

(d) multiplicative identify

5. a. Let x = the number

$2x + 16 = 86$

$2x = 70$

$x = 35$

$2.35 + 16 = 86$

$70 + 16 = 86$

$86 = 86$

b. Let w = weight in pounds of a cubic foot of gold.

$2w - 196 = 2000$

$2w = 2196$

$w = 1098$ lbs.

$2.1098 - 196 = 2000$

$2196 - 196 = 2000$

$2000 = 2000$

ADDITIONAL REFERENCES

Beckenbach, Bellman: An Introduction to Inequalities. New Mathematical Library, New York: Random House, Inc., 1963.

DeVenney; McCraig; Stenzel. Secondary School Mathematics, Special Edition. The Board of Trustees of the Leland Stanford Junior University, 1970.

Dressler, Isadore. Preliminary Mathematics. Amsco School Publications, Inc., New York: 1965.

Eason, Oliver W. Graphing Pictures. Portland, Maine: J. Weston Walsh, 1966.

Estes, Kerber. The New Mathematics: Algebra. Wichita, Kansas: McCormick-Mathers Publishing Co., 1966.

Herrick; Zartman; Conrow. Modern Mathematics for Achievement, Book 2. Atlanta: Houghton Mifflin Co., 1967. Excellent book for student working individually.

Jones; Clark; Potter. Learning to Compute. New York: Harcourt, Brace and World, Inc., 1967.

L S I Contemporary Mathematics Kit. California Test Bureau, McGraw-Hill Book Co., Del Monte Research Park, Monterrey, California.

Niven, Ivan. Numbers: Rational and Irrational. New Mathematical Library, Random House, Inc., New York: The L. W. Singer Co., 1961.

Ore, Oystein. Graphs and Their Uses. New Mathematical Library, New York: Random House, Inc., 1963.

Stenzel, Gilman. Mathematics for Today. Cincinnati, Ohio: McCormick-Mathers Publishing Co., Inc., 1971.

Weber, Rose; Weber, Ruth. The New Mathematics 7. Wichita, Kansas: McCormick-Mathers Publishing Co., 1964.

_____. The New Mathematics 8. Wichita, Kansas: McCormick-Mathers Publishing Co., 1964. Excellent approach to all areas covered in the outline.

Wohlfort, Sheridan. Investigating Mathematical Ideas, Book C. New York: Holt, Rinehart and Winston, Inc., 1969. Supplementary practice.

_____. Investigating Mathematical Ideas, Book D. Ne York: Holt, Rinehart and Winston, Inc., 1969. Supplementary practice.